

# A note on brane/flux annihilation and dS vacua in string theory

Xiao-Jun Wang\*

*Interdisciplinary Center for Theoretical Study*

*University of Science and Technology of China*

*AnHui, HeFei 230026, China*

Neng-Chao Xiao

*Department of Modern Physics*

*University of Science and Technology of China*

*AnHui, HeFei 230026, China*

## Abstract

We reconsider the dynamics of  $p$  anti-D3 branes inside the Klebanov-Strassler geometry, in which  $M$  units of R-R 3-form flux and  $K$  units of NS-NS 3-form flux are presented in deformed conifold. We find that anti-D3 branes blow up into a spherical D5-brane at weak string coupling via quantum tunnelling. The D5-brane can be either stable or unstable, depending on number of background flux. The nucleation rate of D5-brane is suppressed by  $\exp\{-Mp^2\}$ . The classical mechanically the evolution of unstable D5-brane annihilates one unit of R-R flux and ends with  $(K - p)$  D3-branes. This observation is consistent with one by Kachru, Pearson and Verlinde, who shew that anti-D3 branes in KS geometry can blow up into a spherical NS5 brane at strong string coupling, because NS5-brane is lighter than D5-brane at strong string coupling. We also argue that the system can end with a meta-stable dS vacuum by fine tuning of number of background flux.

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\*Electronic address: wangxj@ustc.edu.cn

## I. INTRODUCTION

There has recently been a great deal of interest on the KKLT model[1] since this model provides an explicit realization on de Sitter (dS) vacua in string theory and is consistent with stabilization of various moduli (in particular, including volume modulus). KKLT, following earlier works on flux compactification in IIB theory[2, 3, 4], noted that the volume modulus can be fixed by considering some non-perturbative effects (e.g., the gluino condensation). While the flux compactification yields supersymmetric AdS vacua in  $\mathcal{N} = 1$  supergravity. This AdS minimum may be uplifted to non-supersymmetric dS one by introducing anti-D3 branes into Klebanov-Strassler (KS) geometry[5], which is warped background described by  $M$  units of R-R 3-form flux and  $K$  units of NS-NS 3-form flux presented in the deformed conifold. The key point is that there is extra energy contribution from tension of  $\overline{\text{D3}}$ -branes and extra fluxes. Moreover, the dynamics of  $\overline{\text{D3}}$ -branes is also important because the potential of world-volume scalar may in general contribute a negative energy.

The dynamics of  $p$   $\overline{\text{D3}}$ -branes ( $p \ll K, M$ ) in KS geometry has been studied by Kachru, Pearson and Verlinde (KPV) [6] in strong string coupling region. A small negative minimum of world-volume scalar potential was shown to correspond to a non-supersymmetric NS5-brane “giant graviton” configuration. In other words, the  $\overline{\text{D3}}$ -branes are expanded into a spherical NS5-brane due to the presence of background NS-NS flux, according to the effect first observed by Myers[7]. The NS5-brane is quantum mechanically unstable via tunnelling to a nearby supersymmetric vacuum. The decay rate is exponentially suppressed. Consequently the meta-stable dS vacuum has longer life and world-volume scalar potential only slightly corrects dS minimum in this regime.

The purpose of this present note is to address natural questions that what happens in weak string coupling region, and what role the R-R flux plays. We expect that  $\overline{\text{D3}}$ -branes may be expanded into a non-supersymmetric D5-brane “giant graviton” configuration by R-R flux, as similar as NS5-brane by NS-NS flux. The D5-brane should be nucleated at weak string coupling because in this region it is lighter than NS5 and is easier to be formed. At the first glance we should worry whether the Myers’ effect is induced by R-R flux since  $\overline{\text{D3}}$ -branes in KS background are quickly driven to the end of the throat (or the apex of the deformed conifold) [6], where  $\star F_{(3)}$  disappears. We notice, however, that the apex of the conifold is fuzzy due to non-Abelian effect of world-volume scalar on  $\overline{\text{D3}}$ -branes. In other

words, we can not say where is exact location of the apex. In this sense the Myers' effect is indeed induced by R-R flux. Consequently  $\overline{\text{D3}}$ -branes blow up into a spherical D5-brane.

We shall show that the dynamical process to nucleate spherical D5-brane is in essential different from one to form NS5-brane. It is classically impossible to nucleate D5-brane from  $\overline{\text{D3}}$ -branes without extra initial conditions input (e.g., opposite motion among of  $\overline{\text{D3}}$ -branes). Quantum mechanically, however,  $\overline{\text{D3}}$ -branes can tunnel to a D5-brane “giant graviton” configuration. The tunnelling rate is exponentially suppressed if the number of  $\overline{\text{D3}}$ -branes is larger. Furthermore, D5-brane after quantum birth can be classically either stable or unstable, depending on the numbers of background fluxes. The evolution of unstable D5-brane will annihilate one unit of R-R flux and end with  $(K - p)$  D3-branes. Geometrically, the KS background removes the singularity of the conifold by blowing up a three-sphere at the apex of the conifold<sup>1</sup>. The KPV's NS5-brane is further nucleated by distributing  $\overline{\text{D3}}$ -branes on a two-sphere inside that  $S^3$ . The spherical D5-brane, by contrast, is nucleated by distributing  $\overline{\text{D3}}$ -branes on  $S^2$  transverse to the  $S^3$ .

This note is organized as follows: In section 2, We give a brief review on embedding KS geometry into F-theory compactification. The dynamics of a few of  $\overline{\text{D3}}$ -branes probing in KS background is considered in section 3. In section 4 we compute rate of the quantum tunnelling to nucleate a D5-brane “giant graviton” configuration from  $\overline{\text{D3}}$ -branes. Section 5 lists a summary and some discussions, including the discussion on dS vacua in our model.

## II. EMBEDDING KLEBANOV-STRASSLER GEOMETRY INTO F-THEORY COMPACTIFICATION

Klebanov-Strassler(KS) background[5] is vacuum solution IIB supersgravity. It is produced by placing  $N$  D3-branes and  $M$  D5-branes wrapping on a collapsing supersymmetric two-cycle at the apex of the deformed conifold defined by

$$\sum_{i=1}^4 z_i^2 = \varepsilon^2, \quad (1)$$

in which the singularity of the conifold is removed through the blowing-up of the  $S^3$  of  $T^{1,1}$ . Here  $\varepsilon$  controls the size of the  $S^3$  and has dimensions of length  $3/2$ . At the end of evolution

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[14] The conifold can be described as a cone over the space  $T^{1,1}$ , which can be topologically thought of as  $S^3 \times S^2$ .

the D3-branes are dissolved and their charge is carried by R-R and NS-NS 3-form fluxes. Because  $\overline{\text{D3}}$ -branes will be quickly driven to the end of the throat when they are put into KS geometry[6], we only focus on the geometry near the apex of KS background<sup>1</sup> [8] :

$$\begin{aligned}
ds^2 &\xrightarrow{\tau \rightarrow 0} \frac{\varepsilon^{4/3}}{\sqrt{(a_0 - a_1\tau^2 + a_2\tau^4)} g_s M l_s^2} dx_a dx_a + \sqrt{a_0 - a_1\tau^2 + a_2\tau^4} g_s M l_s^2 \left\{ d\tau^2 + (g^5)^2 \right. \\
&\quad \left. + 2(g^3)^2 + 2(g^4)^2 + \frac{1}{2}\tau^2[(g^1)^2 + (g^2)^2] \right\}, \\
F_{(3)} &\xrightarrow{\tau \rightarrow 0} 2\sqrt{3} M l_s^2 \{ g^5 \wedge g^3 \wedge g^4 + \frac{\tau^2}{12} g^5 \wedge g^1 \wedge g^2 + \frac{\tau}{6} g^5 \wedge (g^1 \wedge g^3 + g^2 \wedge g^4) \}, \\
H_{(3)} &\xrightarrow{\tau \rightarrow 0} 2\sqrt{3} g_s M l_s^2 \{ d\tau \wedge (\frac{1}{3} g^3 \wedge g^4 + \frac{\tau^2}{4} g^1 \wedge g^2) + \frac{\tau}{6} g^5 \wedge (g^1 \wedge g^3 + g^2 \wedge g^4) \}, \\
\mathcal{F}_{(5)} &= B_{(2)} \wedge F_{(3)} \xrightarrow{\tau \rightarrow 0} \frac{4g_s M^2 l_s^4}{3} \tau^3 g^1 \wedge g^2 \wedge g^3 \wedge g^4 \wedge g^5, \\
\star F_{(3)} &\xrightarrow{\tau \rightarrow 0} \frac{2\sqrt{3}\varepsilon^{8/3}}{a_0 g_s^2 M l_s^2} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^2 \wedge \{ d\tau \wedge (\frac{1}{3} g^3 \wedge g^4 + \frac{\tau^2}{4} g^1 \wedge g^2) \\
&\quad + \frac{\tau}{6} g^5 \wedge (g^1 \wedge g^3 + g^2 \wedge g^4) \},
\end{aligned} \tag{2}$$

together with constant dilation field, where the constants  $a_0 \simeq 0.718$ ,  $a_1 = 3^{-4/3}$  and  $a_2 = 2^{1/3}/3^{7/3}$ ,  $g^i$  ( $i = 1, \dots, 5$ ) are one-form basis on  $T^{1,1}$ [9],

$$\begin{aligned}
g^1 &= -\frac{1}{\sqrt{2}}(\sin \theta_1 d\phi_1 + \cos \psi \sin \theta_2 d\phi_2 - \sin \psi d\theta_2), \\
g^2 &= \frac{1}{\sqrt{2}}(d\theta_1 - \sin \psi \sin \theta_2 d\phi_2 - \cos \psi d\theta_2), \\
g^3 &= -\frac{1}{\sqrt{2}}(\sin \theta_1 d\phi_1 - \cos \psi \sin \theta_2 d\phi_2 + \sin \psi d\theta_2), \\
g^4 &= \frac{1}{\sqrt{2}}(d\theta_1 + \sin \psi \sin \theta_2 d\phi_2 + \cos \psi d\theta_2), \\
g^5 &= d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2.
\end{aligned} \tag{3}$$

The transverse geometry near the apex of deformed conifold is thus the  $S^2$  fibered over the  $S^3$ . In the above set of coordinates the two- and three-cycles are respectively parameterized by

$$\begin{aligned}
S^2 : \quad \phi_1 &= -\phi_2, \quad \theta_1 = (-1)^n \theta_2, \quad \psi = (2n+1)\pi, \\
S^3 : \quad \theta_1 &= \phi_1 = 0,
\end{aligned} \tag{4}$$

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[1] Our expressions on metric and various fluxes are different from one in refs. [5, 8] by a rescale of coordinates,  $x^a \rightarrow 2^{1/6} x^a$ ,  $y_i \rightarrow 48^{1/6} y_i$ .

with  $n$  integer. It is easy to check that the two-cycle defined by the above parameterization is that of minimal volume. It means D5-brane wraps on this cycle is with minimal energy. Hence it is a supersymmetric two-cycle[10].

In order to embedding KS geometry into F-theory compactification, one has to incorporate the tadpole cancellation condition

$$\frac{\chi(X)}{24} = N_{D3} - N_{\overline{D3}} + \frac{1}{2\kappa_{10}^2 T_3} \int_{\mathcal{M}} H_{(3)} \wedge F_{(3)}, \quad (5)$$

where  $T_3$  is the D3-brane tension, and  $\chi(X)$  is the Euler number of the CY fourfold  $X$  that specifies the F-theory compactification.

Dirac quantization implies that those fluxes, integrated over all of the three-cycles of the CY, be integers. From Eq. (2) one see that the KS solution corresponds to the placing of  $M$  units of R-R flux through the  $A$ -cycle spanned by  $g^3, g^4, g^5$ , and  $K$  units of NS-NS flux through the dual  $B$ -cycle<sup>2</sup> spanned by  $g^1, g^2, \tau$ ,

$$\begin{aligned} \frac{1}{4\pi^2} \int_A F_{(3)} &= M, \\ \frac{1}{4\pi^2} \int_B H_{(3)} &= -K. \end{aligned} \quad (6)$$

In terms of choosing  $M$  and  $K$  to let  $MK = \chi/24$ , D3-brane charge is conserved without extra D3-brane inserted.

### III. DYNAMICS OF THE PROBE $\overline{D3}$ -BRANES

Let us assume that there is little mismatch between Euler number of CY fourfold and the numbers of fluxes, e.g.,  $MK - p = \chi/24$ . Then two side of tadpole condition (6) has to be balanced by introduce  $p$  anti-D3 branes, which is driven to the end of throat of KS geometry. The characteristic size of the geometry is order  $\sqrt{g_s M} l_s$ , while the backreaction from the  $p$   $\overline{D3}$ -branes can be estimated to extend over a region of order  $\sqrt{g_s p} l_s$ . Hence the distortion of KS geometry due to presence of the  $\overline{D3}$ -branes can be ignored as long as  $p \ll M$ [6].

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[2] In this type of scheme of compactification, KS geometry is cut and glued smoothly with a compacted manifold[11]. Hence radius coordinate  $\tau$  varies in finite region.

### A. World-volume action

The low energy dynamics of  $N$  coincident anti-Dp branes is described by the following non-Abelian DBI action[7, 12]

$$S_{BI} = -T_p \int d^{p+1} \xi \text{STr} \left( e^{-\phi} \sqrt{-\det(P[E_{ab} + E_{ai}(Q^{-1} - \delta)^{ij} E_{jb}] + 2\pi l_s^2 F_{ab}) \det(Q^i_j)} \right), \quad (7)$$

with

$$E_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu}, \quad Q^i_j = \delta^i_j + 2\pi i l_s^2 [\Phi^i, \Phi^k] E_{kj}, \quad (8)$$

plus corresponding Chern-Simons action

$$S_{CS} = -\mu_p \int \text{STr} \left( P[e^{2\pi i l_s^2 i_\Phi} (\sum_n C^{(n)} e^B)] e^{2\pi l_s^2 F} \right) \quad (9)$$

and supplemented by proper fermionic part. Here  $G_{\mu\nu}$ ,  $B_{\mu\nu}$  and  $C^{(n)}$  are background metric, NS-NS 2-form and R-R  $n$ -form potential respectively.  $P[\dots]$  denotes the pullback of the enclosed spacetime tensors to the worldsheet of D-strings. The transverse scalars  $\Phi^i$  are  $N \times N$  matrices in the adjoint representation of the  $U(N)$  worldsheet gauge symmetry.  $\text{STr}(\dots)$  denotes a symmetrical trace in  $U(N)$  gauge group. Finally, the operator  $i_\Phi$  is defined by

$$i_\Phi i_\Phi C^{(n)} = \frac{1}{(n-2)!} \Phi^{i_1} \Phi^{i_2} C^{(n)}_{i_1 i_2 i_3 \dots i_n} dx^{i_3} \wedge \dots \wedge dx^{i_n}. \quad (10)$$

We consider a two-sphere solution which just corresponds to the  $S^2$  defined by Eq. (4). It is not hard to see that the  $S^2$  with  $n = 0$  is equivalent to impose the condition  $g^3 = g^4 = g^5 = 0$ . In language of matrix model from BI-action, it means to reduce six transverse scalar  $\Phi^i$  to three by imposing certain conditions. This is always possible in search of the static solutions. With the above conditions, the transverse metric,  $\star F_{(3)}$  and  $H_{(3)}$  on two-cycle are reduced by

$$\begin{aligned} d\tau^2 + \frac{1}{2} \tau^2 [(g^1)^2 + (g^2)^2] &\longrightarrow d\tau^2 + \tau^2 d\Omega_2^2 = dy_1^2 + dy_2^2 + dy_3^2, \\ \star F_{(3)} &\longrightarrow \frac{\sqrt{3} \epsilon^{8/3}}{a_0 g_s^2 M l_s^2} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^2 \wedge dy_1 \wedge dy_2 \wedge dy_3. \\ H_{(3)} &\longrightarrow \sqrt{3} g_s M l_s^2 dy_1 \wedge dy_2 \wedge dy_3 \quad \Leftrightarrow \quad B_{(2)} = \frac{g_s M l_s^2}{2\sqrt{3}} \epsilon^{ijk} y_i dy_j \wedge dy_k. \end{aligned} \quad (11)$$

To simplify the world-volume action of D3-branes, we set worldvolume gauge field to zero and take the following (static) gauge:

$$\begin{aligned}\xi_a &= x_a, & a &= 0, 1, 2, 3, \\ y_i &= 2\pi l_s \Phi_i, & i &= 1, 2, 3.\end{aligned}\tag{12}$$

Then inserting background (2), (3) and (11) in non-Abelian action (7) and (9), we can expand lagrangian in powers of  $l_s$ . The resulted low energy effective lagrangian as follows<sup>2</sup>:

$$\begin{aligned}\mathcal{L}/T_3 &= -\frac{\varepsilon^{8/3}}{a_0 g_s^2 M^2 l_s^4} \left[ p + \frac{4\pi^2 l_s^2 a_1}{a_0} \text{Tr} \Phi_i^2 + \frac{16\pi^4 l_s^4 a_1}{a_0} \left( \frac{a_1}{a_0} - \frac{a_2}{a_1} \right) \text{Tr} (\Phi_i \Phi^i)^2 \right] \\ &\quad - 2\pi^2 l_s^2 \varepsilon^{4/3} \eta_{ab} \text{Tr} (\partial^a \Phi_i \partial^b \Phi_i) + \pi^2 \varepsilon^{8/3} \text{Tr} ([\Phi_i, \Phi_j]^2) \\ &\quad - \frac{2i\pi^2 \varepsilon^{8/3}}{\sqrt{3} a_0 g_s M l_s} \epsilon_{ijk} \text{Tr} (\Phi^i \Phi^j \Phi^k) - \frac{i}{3} 4\pi^2 g_s l_s [\star F_{(3)}]_{0123ijk} \text{Tr} (\Phi^i \Phi^j \Phi^k) + \dots\end{aligned}\tag{13}$$

There are two notable new features in the above lagrangian: 1) The last two terms in the first line are from warped factor of background. 2) The first term in the third line is from NS-NS  $B$ -field which plays a role in worldvolume action through eq. (7). It identifies to one from background R-R flux (the last term in the third line), due to “no-force” condition between D3-brane and imaginary self-dual flux background[6].

## B. Static solutions

With the normalization,

$$\Phi_i \rightarrow \frac{\Phi_i}{2\pi l_s \varepsilon^{2/3} \sqrt{T_3}}, \quad T_3 = \frac{1}{g_s (2\pi)^3 l_s^4},$$

the effective scalar potential for the present problem is

$$V(\Phi) = V_0 + \frac{m^2}{2} \text{Tr} \Phi_i^2 - \frac{\pi g_s}{2} \text{Tr} ([\Phi_i, \Phi_j]^2) - \frac{2\pi g_s \lambda}{p^2 - 1} \text{Tr} (\Phi_i \Phi^i)^2 + \frac{2if\epsilon_{ijk}}{3} \text{Tr} (\Phi^i \Phi^j \Phi^k),\tag{14}$$

with

$$\begin{aligned}V_0 &= \frac{p T_3 \varepsilon^{8/3}}{a_0 g_s^2 M^2 l_s^4} & m^2 &= \frac{2a_1 \varepsilon^{4/3}}{a_0^2 g_s^2 M^2 l_s^4}, & f &= \frac{\sqrt{6\pi} \varepsilon^{2/3}}{2a_0 \sqrt{g_s} M l_s^2}, \\ \lambda &= \frac{\kappa^2 (p^2 - 1)}{g_s^2 M^2} & \kappa^2 &= \frac{4\pi^2 a_1}{a_0^2} \left( \frac{a_2}{a_1} - \frac{a_1}{a_0} \right) \simeq 1.736 \simeq \sqrt{3}.\end{aligned}\tag{15}$$

To find the extreme of the potential (15), we obtain the solution:

$$\Phi_i = \frac{m^2}{f} u J_i, \quad u = u_0 = \frac{1}{2x} (1 \pm \sqrt{1 - 4x}),\tag{16}$$

where  $x$  is defined by

$$x = \frac{2\pi g_s(1-\lambda)m^2}{f^2} = \frac{8a_1}{3}(1-\lambda), \quad (17)$$

$J_i$  are generators of N-dimensional representation of the  $SU(2)$  group

$$[J_i, J_j] = i\epsilon_{ijk}J_k. \quad (18)$$

According to Eq. (16) we have to impose  $x \in [0, 1/4]$  to admit non-trivial solution.

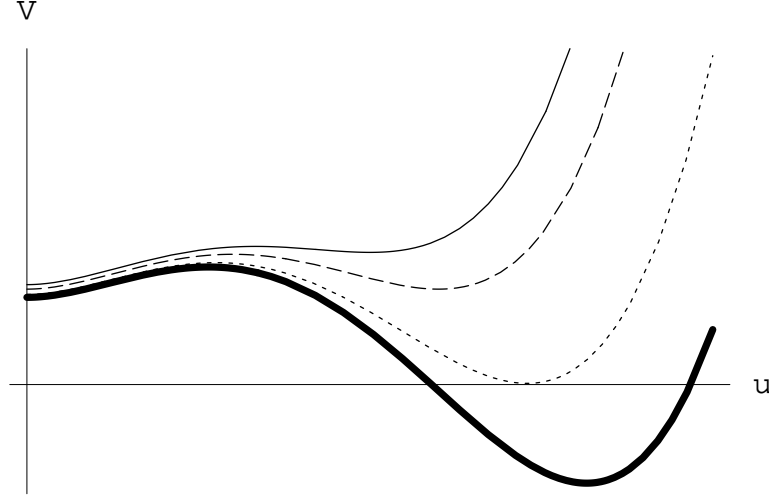


FIG. 1: The curve of effective potential  $V(u)$  for diverse values of parameter  $x$ . The solid line, dash line, dot line and bold line corresponds to  $x = 0.24, 2/9, 0.2, 0.18$  respectively, where local minimum at  $u = u_0$  has the properties  $V(u_0) > V_0$ ,  $V(u_0) = V_0$ ,  $V(u_0) = 0$  and  $V(u_0) < 0$  respectively.

It is useful to rewritten effective potential (14) by new variable  $u$

$$V(u) = V_0 + \frac{p(p^2-1)m^6}{4f^2} \left\{ \frac{1}{2}u^2 - \frac{1}{3}u^3 + \frac{x}{4}u^4 \right\}. \quad (19)$$

In figure 1 we draw the curve of  $V(u)$  for several different values of  $x$ . We see two solutions presented in Eq. (16) respectively correspond to a local maximum and a local minimum:

$$\begin{aligned} V_{\text{Imax}} &= V_0 - \frac{p(p^2-1)m^6}{96x^3f^2} [1 - 6x + 6x^2 - (1-4x)^{3/2}] \geq V_0, & \text{for } x \in [0, \frac{1}{4}], \\ V_{\text{Imin}} &= V_0 - \frac{p(p^2-1)m^6}{96x^3f^2} [1 - 6x + 6x^2 + (1-4x)^{3/2}] \leq V_0, & \text{for } x \in [0, \frac{2}{9}]. \end{aligned} \quad (20)$$

One get, therefore, the condition to admit a non-trivial and stable solution,

$$0 \leq x \leq \frac{2}{9} \quad \Leftrightarrow \quad 1 - \frac{1}{12a_1} \leq \frac{\kappa^2(p^2-1)}{g_s^2 M^2} \leq 1. \quad (21)$$



Because  $\kappa$  is a constant of order one, and  $1 \ll p^2 \ll M^2$ , we obtain  $g_s \sim p/M \ll 1$ . In other words, this solution is valid in region of weak string coupling, as we expected. Furthermore, in order to make the expansion in effective lagrangian (13) be valid, one has

$$\frac{l_s m^2 u}{2\pi l_s \epsilon^{2/3} \sqrt{T_3} f} \ll 1 \quad \Leftrightarrow \quad x g_s M \gg 1 \quad \Leftrightarrow \quad x p \gg 1.$$

This condition in general can be satisfied.

### C. Physical explanation

Geometrically, such the solution represents the fuzzy two-sphere. The radius of two-sphere can be measured as

$$R \sim \sqrt{g_s M} l_s^2 \left( \frac{\text{Tr} \Phi_i^2}{p} \right)^{1/2} \sim \frac{l_s p}{\sqrt{g_s M}} \sim \sqrt{g_s M} l_s. \quad (22)$$

It is just typical scale of background. It is well known that the local minimum presented in Eq. (20) represents vacuum corresponding to a spherical D5-brane configuration with topology  $R^3 \times S^2$ . This spherical D5-brane does not carry net D5-brane charge. Rather, it would be an “electric-dipole”-like configuration due to the separation of oppositely orientation of  $\overline{\text{D3}}$ -brane on surface of fuzzy  $S^2$ . From the viewpoint of dual D5-brane, the process is described by  $p$   $\overline{\text{D3}}$ -branes bounding on D5-brane. At the end of evolution the  $\overline{\text{D3}}$ -brane dissolve into the  $U(1)$  flux along two-sphere which is just world-volume gauge field strength on D5-brane. While the pull back of background NS-NS  $B$ -field onto world-volume of D5-brane keeps the gauge symmetry on world-volume. Following steps presented in [6, 7], it is not hard to prove that the effective potential (19) can be obtained from world-volume Abelian action of D5-brane with a constant gauge field strength,  $F_{\theta\phi} = p \sin \theta$  with  $\theta, \phi$  angle coordinates of  $S^2$ . We do not repeat the details of calculation here.

From figure 1 we find two interesting new features on our model: 1) Classically D5-brane can not be nucleated from  $\overline{\text{D3}}$ -brane if without extra initial conditions, which in general breaks supersymmetry and makes system be unstable. Quantum mechanically, however, the vacuum corresponding to  $\overline{\text{D3}}$ -brane can tunnel to one corresponding to spherical D5-brane. The tunnelling rate will be computed in next section. 2) From figure 1 we see that the total energy of system is lowered with decrease of  $x$  and can be even negative. Of course, the negative energy is unphysical. It just implies that spherical D5-brane is ripped into D5- $\overline{\text{D5}}$

pair by background flux when size of fuzzy  $S^2$  grows to a critical value. The anti-D5 brane is quickly driven to the apex of KS geometry and annihilate one unit of background R-R flux. The D5-brane, however, carries  $(K-p)$  D3-brane charge and gets no force from background. Rather, it receives a small initial velocity from  $\overline{\text{D5}}$  before  $\overline{\text{D5}}$ -brane disappears and hence slowly moves to the apex of KS geometry. At the apex it dissolves into  $(K-p)$  D3-branes.

#### IV. VACUUM TUNNELLING

We now turn to compute the nucleation rate of spherical D5-brane which is generated by quantum tunnelling without extra initial conditions. It is standard way to find classical solution of Euclidean action:

$$S_E = \frac{p(p^2-1)m^4}{4f^2} \int dr r^3 \left\{ \frac{1}{2}u'^2 + \frac{4f^2V_0}{p(p^2-1)m^4} + m^2 \left( \frac{1}{2}u^2 - \frac{1}{3}u^3 + \frac{x}{4}u^4 \right) \right\}, \quad (23)$$

where  $u' = du/dr$  with  $r$  radial coordinate of  $R^4$ . The equation of motion of Euclidean version reads off

$$\begin{aligned} u'' &= m^2 u(1-u+xu^2), \\ \Rightarrow u'^2 &= m^2 u^2 \left( 1 - \frac{2}{3}u + \frac{x}{2}u^2 \right) + c_0. \end{aligned} \quad (24)$$

We look for the lowest energy solution whose kinetic energy vanishes at meta-stable vacuum  $V(u=0)$ , so that we have  $c_0 = 0$ . The full solution can be obtained explicitly:

$$u(r) = \frac{3}{1 + \cosh(mr) - \sqrt{\frac{9x}{2}} \sinh(mr)}. \quad (25)$$

It looks to be similar to the instanton solution proposed in [13]. The tunnelling occurs between  $u=0$  and  $u=u_* = \frac{2}{3x}(1 - \sqrt{1 - \frac{9x}{2}})$ . Then the system classically rolls down to more stable vacuum at  $u=u_0 = \frac{1}{2x}(1 + \sqrt{1 - 4x})$ . Using the solution (25) we have

$$\begin{aligned} u=0 &\Leftrightarrow r \rightarrow \infty, \\ u_* = \frac{2}{3x} \left( 1 - \sqrt{1 - \frac{9x}{2}} \right) &\Leftrightarrow r_* = \frac{1}{m} \ln \frac{1 + \sqrt{\frac{9x}{2}}}{\sqrt{1 - \frac{9x}{2}}}. \end{aligned} \quad (26)$$

In figure 2 we plot the Euclidean trajectories  $u(r)$  for diverse values of  $x$ . It implies that the tunnelling starts from vacuum at  $u=0$  at large  $r$ . The scalar field stays that meta-stable vacuum until  $r$  close to  $r_*$ , where it quickly rolls down to more stable vacuum at  $u=u_0$ .

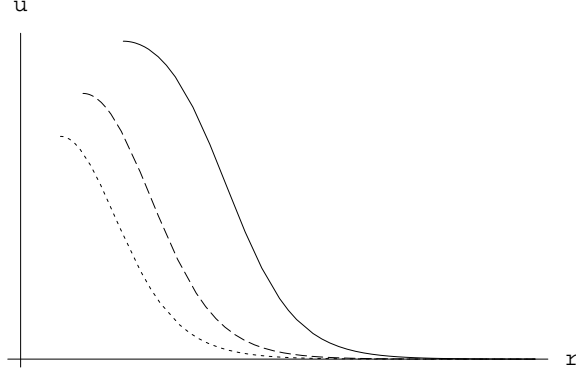


FIG. 2: The Euclidean trajectories  $u(r)$  for diverse values of parameter  $x$ . The solid line, dash line and dot line correspond to  $x = 2/9$ ,  $0.2$ ,  $0.15$  respectively, where the trajectories end with  $r = r_*(x)$ .

Substituting the solution (25) into Euclidean action (23) one has

$$S_E = \frac{p(p^2 - 1)m^2}{4f^2} \int_{\tilde{r}_*}^{\infty} d\tilde{r} \tilde{r}^3 u^2 \left(1 - \frac{2}{3}u + \frac{x}{2}u^2\right) - \frac{V_0}{4m^4} \tilde{r}_*^4, \quad (27)$$

with  $\tilde{r} = mr$ . Using definition on  $f$ ,  $m^2 V_0$  in Eq. (15) and (17), we obtain that the nucleation rate of D5-brane is about

$$\exp \left( -\frac{a_1}{6\pi g_s} p(p^2 - 1) \mathcal{T}(x) \right) \simeq \exp \left( -\frac{a_1 M}{8\pi \kappa} p^2 \sqrt{1 - \frac{3x}{8a_1}} \mathcal{T}(x) \right), \quad (28)$$

with

$$\mathcal{T}(x) = \int_{\tilde{r}_*}^{\infty} d\tilde{r} \tilde{r}^3 u^2 \left(1 - \frac{2}{3}u + \frac{x}{2}u^2\right) - \frac{3a_0^3 \kappa^2}{64\pi^2 a_1^2 (1 - 3x/(8a_1))} \tilde{r}_*^4. \quad (29)$$

In figure 3 we show the curve of  $\mathcal{T}(x)$ . Hence the nucleation rate of D5-brane is strong suppressed with the number of  $\overline{D3}$ -branes increasing.

## V. SUMMARY AND DISCUSSIONS

We have refilled that the dynamics of  $p$   $\overline{D3}$ -branes probe the KS geometry in which  $M$  units R-R flux and  $K$  units NS-NS flux are presented. An interesting new feature is that warped factor plays crucial role to world-volume action of  $\overline{D3}$ -branes. In other words, it imposes a strong constraint among  $p$ ,  $M$  and  $g_s$  in order that the system has nontrivial static solution. In weak coupling region, i.e.,  $g_s \sim p/M \ll 1$ , that nontrivial static solution is a more stable nonsupersymmetric vacuum which corresponds to D5-brane “giant graviton” configuration. Without any extra initial condition, the nonsupersymmetric vacuum

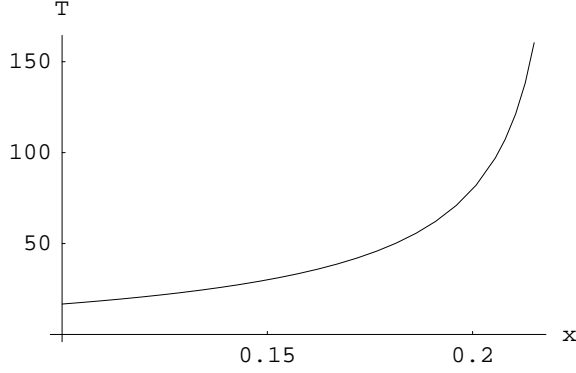


FIG. 3: The curve of  $\mathcal{T}(x)$ .

corresponding to  $\overline{D3}$ -branes transfers to that spherical D5-brane vacuum by quantum tunnelling. The tunnelling rate, however, gets strong suppression with growth of the number of  $\overline{D3}$ -branes. The spherical D5-brane can classically be either stable or unstable. At the end of evolution, the unstable D5-brane annihilates one unit R-R flux and results in  $K - p$  D3-branes.

It is very interesting to compare the evolution of spherical D5-brane with one of spherical NS-brane considered by KPV [6]:

$$\begin{array}{ccccccc}
 g_s \ll 1 : & p \overline{D3} & \longrightarrow & D5 (S^2 \text{ inside } A\text{-cycle}) & \longrightarrow & \text{stable} & \\
 & & & & & \longrightarrow & (K - p) D3 \\
 & & & \text{QT} & & \text{CE} & \\
 g_s \gg 1 : & (M - p) D3 & \longleftarrow & NS5 (S^2 \text{ inside } B\text{-cycle}) & \longleftarrow & p \overline{D3} & 
 \end{array}$$

where “QT” and “CE” denote “quantum tunnelling” and “classical evolution” respectively. Two processes look like to be inverse each other in brane/flux transmutation and give a good match under  $S$ -duality.

Now let us assume that, without any insertion of D3- and  $\overline{D3}$ -branes, we have embedded KS geometry into F-theory compactification. The tadpole condition is satisfied by taking  $\chi(X)/24 = MK$ . After all of moduli are stabilized, we end with a supersymmetric AdS vacuum<sup>3</sup> We want to uplift the AdS vacuum to dS one by adding few  $\overline{D3}$ -branes while the topology of CY fourfold  $X$  is not changed. It is possible by adding extra flux simultaneously. For example, we can add one unit R-R flux and  $K$   $\overline{D3}$ -branes without violation of tadpole

[3] The authors of ref.[14] pointed out that there is no known model that stabilizes the volume modulus with fluxes. We however still adopt the viewpoint of KKLT model.

condition. Both of tension of  $\overline{D3}$ -branes and flux contribute to extra vacuum energy which uplift the AdS vacuum to dS one. If  $g_s \sim K/M \ll 1$ ,  $\overline{D3}$ -branes can tunnel to a classically stable spherical D5-brane configuration which corresponding to a more stable dS vacuum<sup>4</sup>.

It can also tunnel to a classically unstable D5-brane configuration which at the end of evolution annihilate one unit R-R flux. In this case we end with the original supersymmetric AdS vacuum again.

A more interesting example is to add two unit extra R-R fluxes and  $2K$   $\overline{D3}$ -branes into background. When

$$\frac{(2K)^2}{g_s^2 M^2} = \frac{1}{\kappa^2} \left(1 - \frac{3x_1}{8a_1}\right), \quad x_1 < 0.2 \quad (30)$$

an unstable spherical D5-brane is nucleated by quantum tunnelling (see Eq. (17) and fig. 1). At the end of evolution it annihilates one unit R-R flux and remain  $K$   $\overline{D3}$ -branes in background. If these residual  $\overline{D3}$ -branes can tunnel to a spherical D5-brane again, we require

$$\frac{K^2}{g_s^2 M^2} = \frac{1}{\kappa^2} \left(1 - \frac{3x_2}{8a_1}\right), \quad x_2 \leq 2/9. \quad (31)$$

Combining Eqs. (30) and (31) we have  $4x_2 - x_1 = 8a_1 \simeq 1.85$ , which can not be satisfied. Hence the residual  $\overline{D3}$ -branes are stable so that the tension  $K$   $\overline{D3}$ -branes and one unit extra R-R flux contribute extra energy to four-dimensional effective potential. In other words, it is possible to end with a more stable dS vacuum in this example.

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[4] From perspective of on-shell string theory, all dS vacua are meta-stable. They must tunnel to supersymmetric vacua when scalar fields become large. This region is beyond our consideration in this present note.

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